Simulating Nucleosynthesis in Tidally Disrupted Stars

HEDP End of Summer Presentation

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Motivation

- Nucleosynthesis
  - Creating heavier isotopes through nuclear fusion
  - Large source of energy in stars ⇒ must be included in hydrodynamic simulations
- Numerically approximated by evolving a *reaction network*
  - Finite set of isotopes and the nuclear reactions that link them
  - More isotopes ⇒ more accurate but more expensive
- Cosmos++: relativistic, radiation-magneto-hydrodynamics multiphysics code for astrophysics applications
  - 7 and 19 isotope networks
  - Tuned to match energy generation in simple problems (complex?)
  - Low isotopic resolution

Goals

- Implement a generalized reaction network in Cosmos++
- Investigate the effect of tidal disruption on stellar composition

Nucleosynthesis Background

Nuclear Reaction Rate

- For binary reactions of the form

$$a + b \rightarrow c + d$$

The nuclear reaction rate is

$$R_{ab} = \rho Y_a Y_b \lambda_{ab}(T)$$

  - ρ: density
  - Y_i: molar abundance of isotope i
  - $\lambda_{ab}(T) = \langle \sigma_{ab} v_{ab} \rangle$: astrophysical reaction rate
    - Averaged over assumed Maxwellian velocity distribution
    - Strong function of temperature (T)

Reaction Network

- Each isotope has a *conservation equation* that describes the evolution of its abundance

Conservation of Isotope i

$$\underbrace{\frac{\mathrm{d}Y_i}{\mathrm{d}t}}_{\text{Change Rate}} = \underbrace{\sum_{j,k} R_{jk}}_{\text{Gain Rate}} - \underbrace{\sum_{j} R_{ij}}_{\text{Loss Rate}}$$

- Number of nucleons is conserved
- Set of conservation equations forms a system of coupled ordinary differential equations

Numerical Nucleosynthesis

- The system is of the form

$$\frac{\mathrm{d}\vec{Y}}{\mathrm{d}t} = f(\vec{Y})$$

  - System of ordinary differential equations
  - Nonlinear
  - Stiff

- Example: linearized, implicit Euler time integration

$$\frac{\vec{Y}^{i+1} - \vec{Y}^{i}}{\Delta t} = f(\vec{Y}^{i+1}) \xrightarrow{TSE} f(\vec{Y}^{i}) + \mathbf{J}(\vec{Y}^{i})(\vec{Y}^{i+1} - \vec{Y}^{i})$$

Need to evaluate $f(\vec{Y}^i)$ and $\mathbf{J}(\vec{Y}^i)$

Constructing a Generalized Network

Generalized Network

- Compute $f(\vec{Y}^i)$ and $\mathbf{J}(\vec{Y}^i)$ for any reaction type (N-body)

$$f(Y_i) = \sum^{\#\text{Reactions}} \rho^{N-1} \frac{c_i}{\prod_k^N c_k!} \lambda(T) \prod_k^N Y_k^{c_k}$$

$$\frac{\partial f(Y_i)}{\partial Y_j} = \sum^{\#\text{Reactions}} \rho^{N-1} \frac{c_i}{\prod_k^N c_k!} \lambda(T) c_j Y_j^{c_j - 1} \prod_{k, k \neq j}^N Y_k^{c_k},$$

  - Decays
  - Any number of reactants and products (including repetitions)

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Astrophysical Reaction Rates, λ

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$$\lambda_{\text{triple alpha}}(T) = 2.9 \times 10^{-16} \left(7.40 \times 10^5 / T_9^{3/2} e^{-1.0663/T_9} + 4.16 \times 10^9 / T_9^{2/3} e^{-13.490/T_9^{1/3} - (T_9/.098)^2} \right)$$

$$\times \left(1 + 0.031 T_9^{1/3} + 8.009 T_9^{2/3} + 1.732 T_9 + 49.883 T_9^{4/3} + 27.426 T_9^{5/3} \right)$$

$$\times \left(1.3 \times 10^2 / T_9^{3/2} e^{-3.3364/T_9} + 2.510 \times 10^7 / T_9^{2/3} e^{-23.570/T_9^{1/3} - (T_9/0.235)^2} \right)$$

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  - Error prone
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  - Difficult to update/maintain
  - Error prone
  - Reevaluating rates wastes CPU cycles
- Limiting factor in implementing a larger network

StarLib Reaction Library

- StarLib Reaction Rate Library (A. Sallaska, 2013)
  - Expansive collection of reactions ($\sim$ 70,000)
  - Tables instead of functions
  - Include experimental uncertainty
- Manually add reactions by specifying reactants and products
- Or automatically pull in reactions where all reactants and products are tracked in the network
- Much easier to manage/maintain reaction rates
- Quickly built 46, 136, 495 isotope networks

Timing Comparison

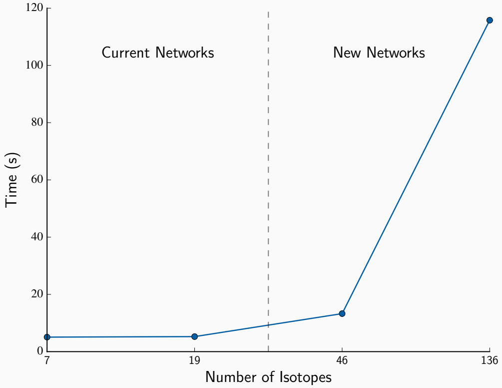

Figure 1: Time to solution in Si burn with $T = 6 \times 10^9\,\mathrm{K}$, $\rho = 1 \times 10^7\,\mathrm{g/cm^3}$. Implicit RK5, 1000 time steps.

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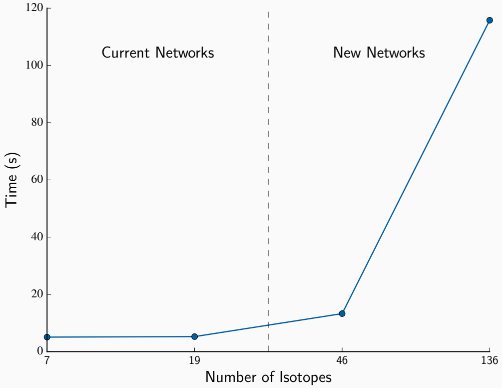

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Generalized networks too slow

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- Post process a few time steps with a larger network to regain isotopic resolution
- Each grid point needs a network solve
- High resolution simulation can have $\sim 10^6$ grid points per time step
  - 2 minutes/burn $\times 10^6$
  - Burn in parallel with MPI

Tidal Disruption

Tidal Disruption of a White Dwarf

- Occurs when a star is sufficiently close to a black hole
- Black hole's gravitational forces break up the star
- White dwarf tidal disruption simulations can be used to investigate properties of intermediate mass black holes
- Cosmos++:
  - Tidal disruption deck (prewritten)
  - 19 isotope network
  - 0.6 $M_\odot$ white dwarf composition set by MESA (Paxton, 2015) stellar evolution simulation
  - Post processed with 136 isotope network

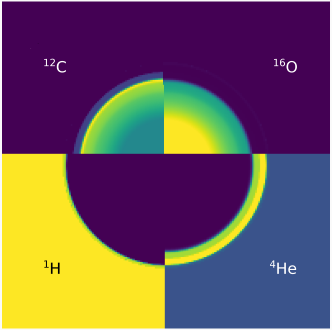
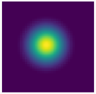
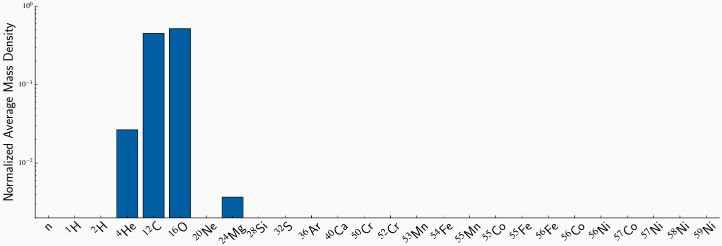


Figure 2: Initial mass fractions computed with MESA.

Tidal Simulation





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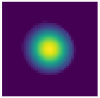

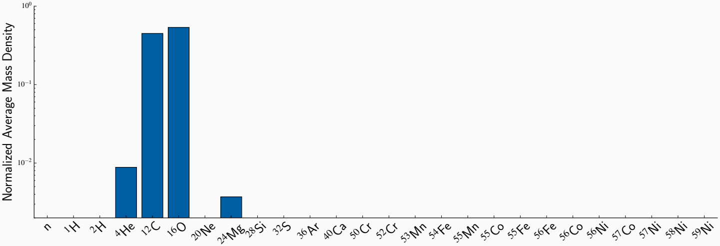

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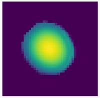

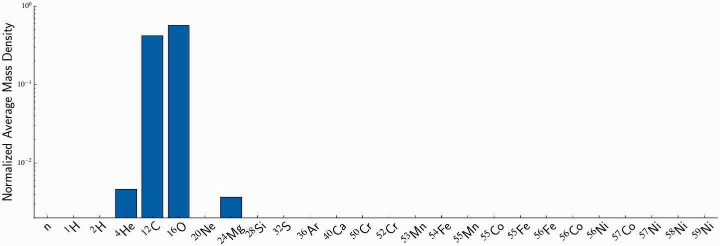

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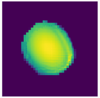

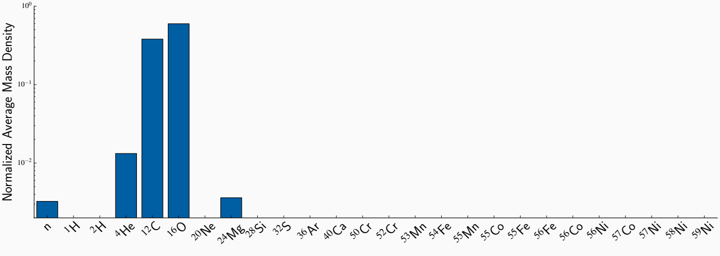

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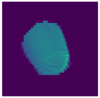

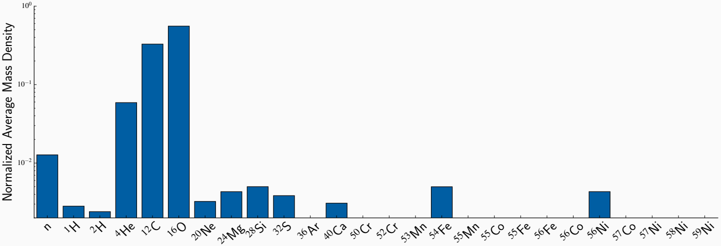

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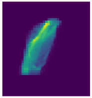

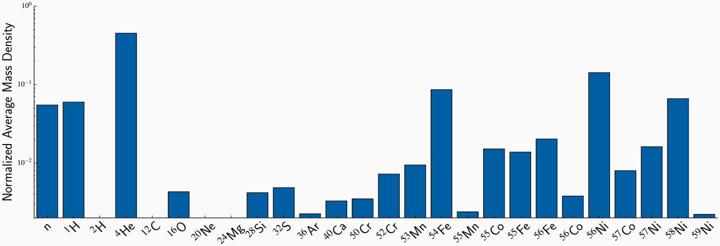

Conclusions

- Cosmos++ now has:
  - A new reaction network interface for creating arbitrarily large networks
  - Modernized reaction rates from StarLib
  - Massively parallel, large network nucleosynthesis post processor
  - Solid foundation for implementing advanced network functionalities
    - Adaptive network size
    - Approximate Jacobian with smaller network's Jacobian
- Future Work
  - Reduce expense of generalized network (too many reactions?)
  - Increase parallel efficiency of post processor (load balancing, GPU?)
  - Compare small network + post processor v. simulation with large network
  - Continue work on tidal simulation

References

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Lawrence Livermore National Laboratory

Hydrostatic Si Burn Test Problem

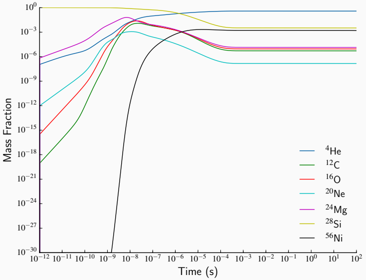

Figure 3: 136 iso network Si burn with $T = 6 \times 10^9$ K, $\rho = 1 \times 10^7$ g/cm^3. Implicit RK5, 1000 time steps.

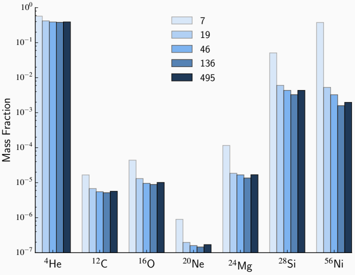

Figure 4: End of evolution iso7 isotopes from 7, 19, 46, and 136 iso networks compared to the 495 iso network from [5].

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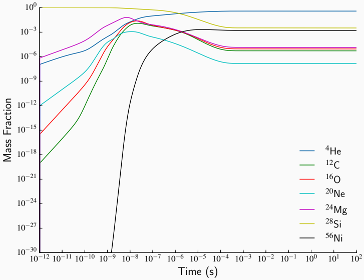

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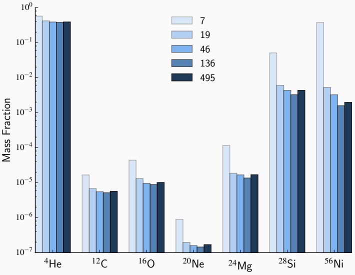

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46 isotope network isn't general enough

Si Burn Energy Generation Comparison

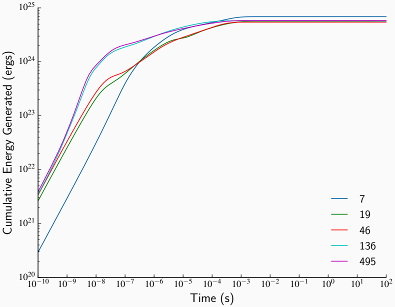

Figure 5: Cumulative energy generated from 7, 19, 46, and 136 iso networks compared to 495 iso network from [5]. Error: 7: 17.8%, 19: 2.62%, 46: 6.62%, 136: 2.14%.

46 iso network isn't general enough to be more accurate than the highly tuned 19 iso network