Simulating Nucleosynthesis in Tidally Disrupted Stars

Samuel Olivier[1], Mentors: Peter Anninos[2], Rob Hoffman[2]

[1]Texas A&M University, [2]Lawrence Livermore National Laboratory

Introduction

- Nucleosynthesis:
  - Creating heavier isotopes through nuclear fusion
  - Large source of energy in stars ⇒ must be included in hydrodynamic simulations of stellar events
  - Numerically evolved through time integration of a *reaction network*
- Reaction Network: a set of isotopes and the reactions that link them together used to approximate nucleosynthesis
- Typically, small reaction networks tuned for energy generation are used to reduce computational expense [1]
- Cosmos++: relativistic, radiation-magneto-hydrodynamics astrophysics code [2]
- Goals
  - Increase isotopic resolution of Cosmos++ simulations by implementing a softwired reaction network
  - Investigate the effect of tidal disruption on stellar composition

Nucleosynthesis Equations

- For binary reactions of the form

$$a + b \leftrightarrows c + d$$

Conserve molar abundance of nuclide i with:

$$\underbrace{\frac{dY_i}{dt}}_{\text{Change Rate}} = \underbrace{\sum_{k,\ell} \rho Y_k Y_\ell \lambda_{k,\ell}}_{\text{Gain Rate}} - \underbrace{\sum_{i,j} \rho Y_i Y_j \lambda_{i,j}}_{\text{Loss Rate}},$$

where Y is molar abundance, ρ density, $\lambda = \langle \sigma v \rangle$ the reaction rate
- Assemble into a system of nonlinear, ordinary differential equations

$$\frac{d\vec{Y}}{dt} = f(\vec{Y})$$

- Implicit, linearized time integration

$$\frac{\vec{Y}^{i+1} - \vec{Y}^{i}}{\Delta t} = f(\vec{Y}^{i+1}) \xrightarrow{\text{TSE}} f(\vec{Y}^{i}) + \mathbf{J}(\vec{Y}^{i})(\vec{Y}^{i+1} - \vec{Y}^{i})$$

- Energy generation:

$$E_{\text{nuclear}} = \sum_{i}^{\#\text{Isotopes}} B_i \Delta Y_i$$

where B_i is the binding energy for isotope i
- Need to evaluate $f(\vec{Y}^i)$ and $\mathbf{J}(\vec{Y}^i)$

Reaction Rates

- Dependent on nuclear cross section, Coulomb potential, quantum mechanical tunneling
- Assume Maxwellian velocity distribution for projectile and target
- Reduces six dimensional relative velocity dependence to just temperature dependence
- Typically fit with power law of temperature and *hardwired* into the code
- StarLib astrophysical reaction rate library [3]
  - Tables instead of functions
  - Includes experimental uncertainties
  - Expansive collection of reactions

Softwired Network

- Generalized function and Jacobian evaluation for N-body reactions

$$f(Y_i) = \sum^{\#\text{Reactions}} \rho^{N-1} \frac{c_i}{\prod_k^N c_k!} \lambda \prod_k^N Y_k^{c_k}$$

$$\frac{df(Y_i)}{dY_j} = \sum^{\#\text{Reactions}} \rho^{N-1} \frac{c_i}{\prod_k^N c_k!} \lambda c_j Y_j^{c_j - 1} \prod_{k, k \neq j}^N Y_k^{c_k},$$

- Automatically pull reactions from StarLib database where all reactants and products are tracked in the network
- Don't need to carefully build networks anymore!

Results

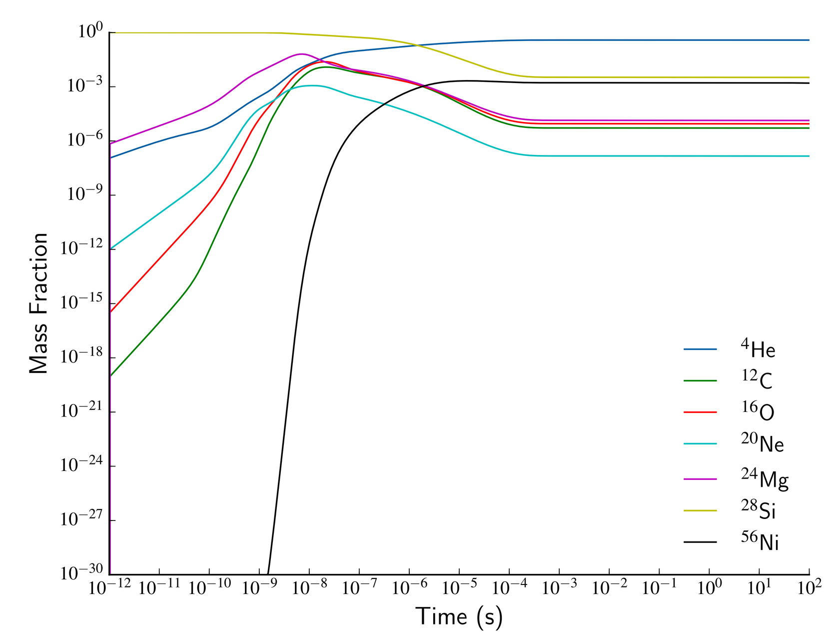

Fig. 1: 136 iso network Si burn with $T = 6 \times 10^9$ K, $\rho = 1 \times 10^7$ g/cm^3. Implicit RK5, 1000 time steps.

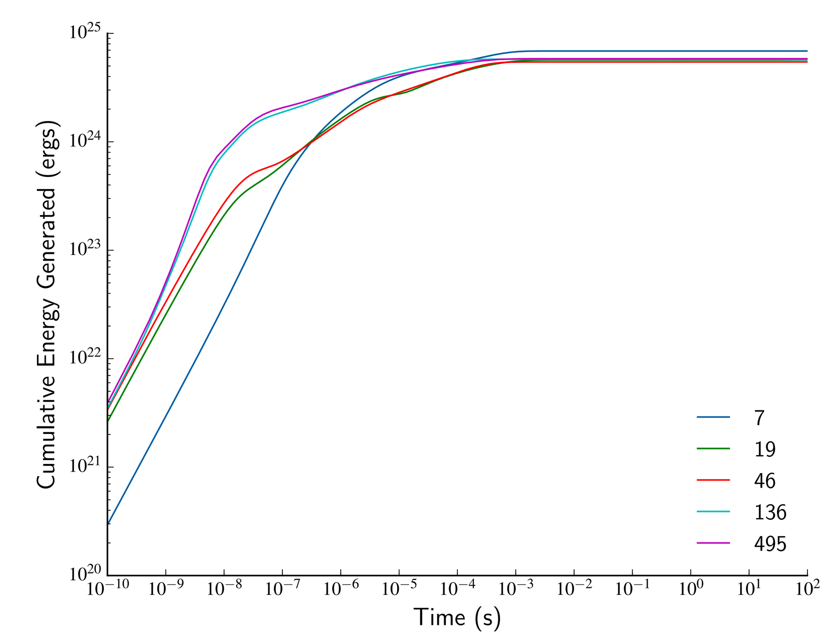

Fig. 2: Cumulative energy generated from 7, 19, 46, and 136 iso networks compared to 495 iso network from [4]. Error: 7: 17.8%, 19: 2.62%, 46: 6.62%, 136: 2.14%.

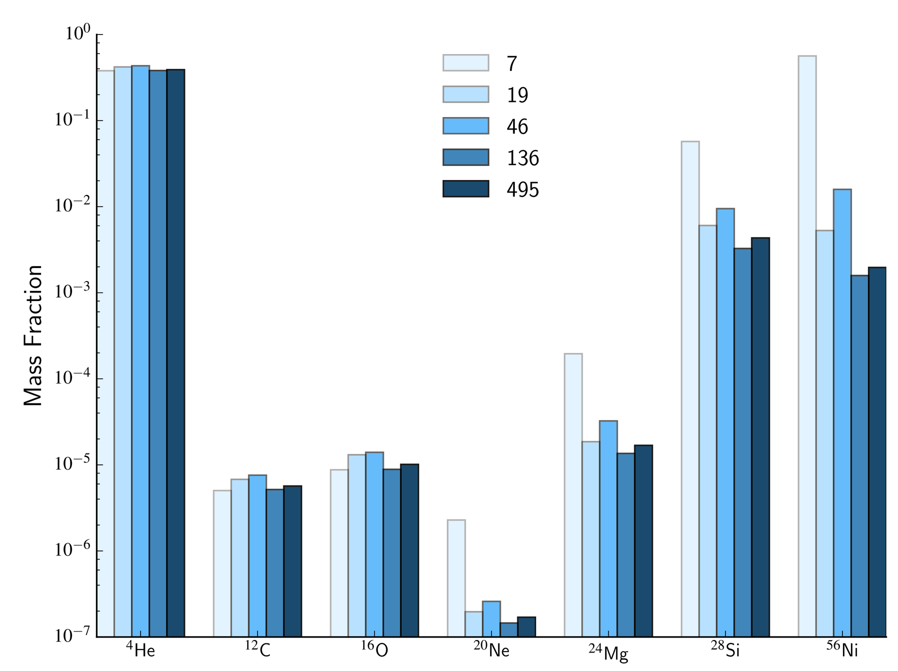

Fig. 3: End of evolution iso7 isotopes from 7, 19, 46, and 136 iso networks compared to the 495 iso network from [4].

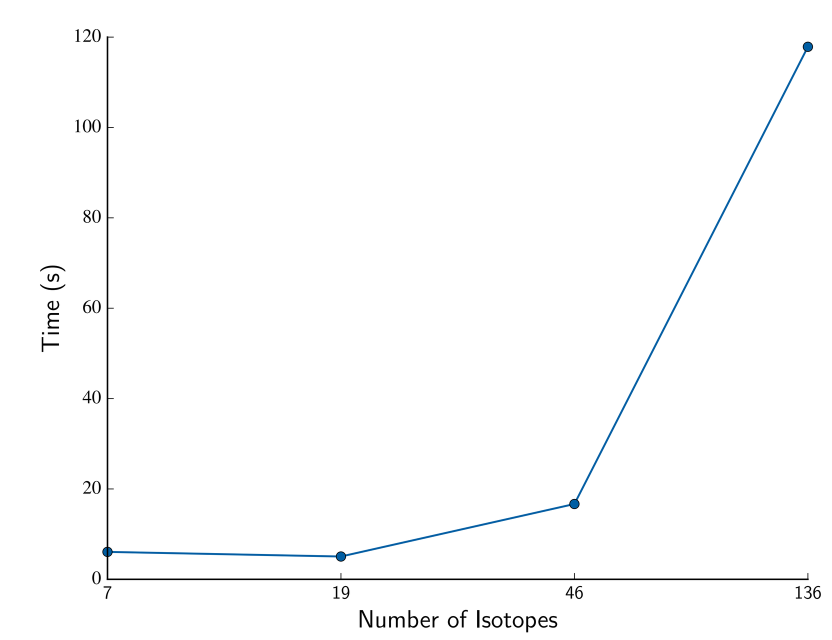

Fig. 4: Time to solution for each network size in the Si burn problem.

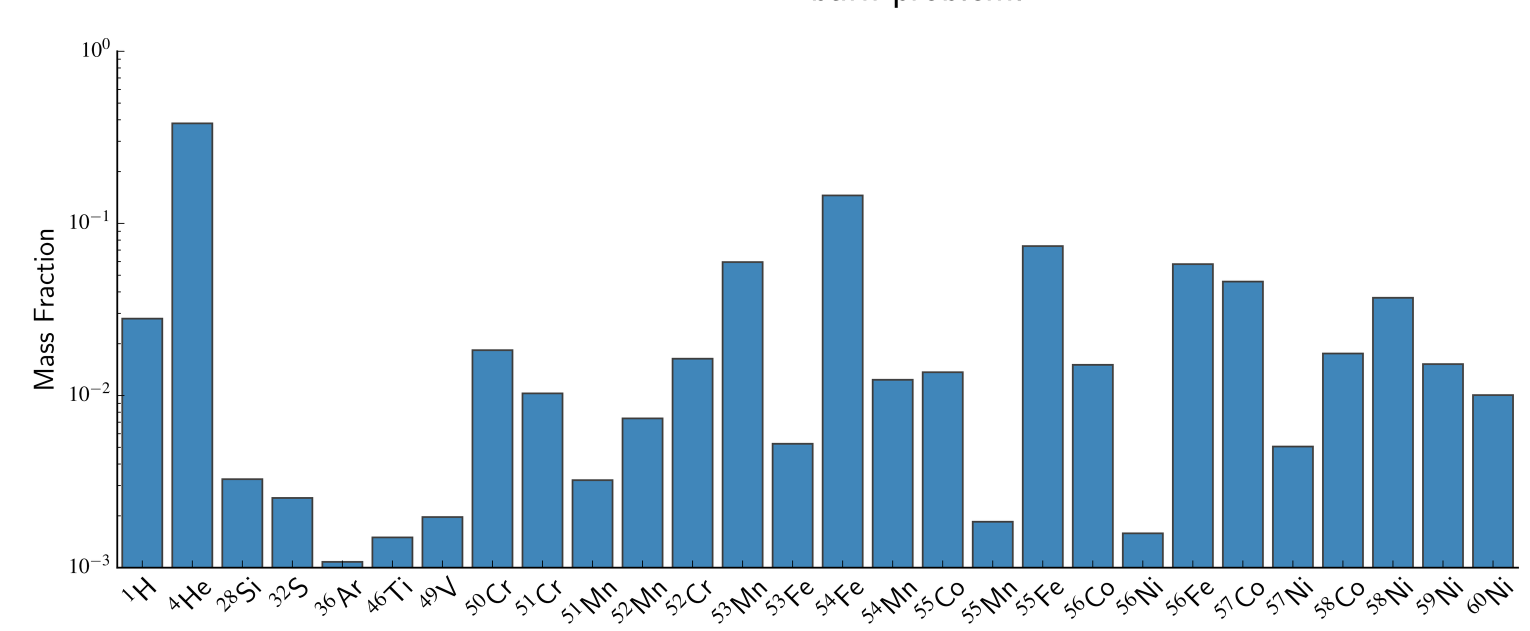

Fig. 5: Mass fractions $> 10^{-3}$ for the 136 iso network.

Nucleosynthesis Post Processor

- Large networks significantly increase wall time
- Small networks can approximate energy generation but do not have high isotopic resolution
- Use small network to approximate energy generation in a hydrodynamic simulation
- Large network hydrostatic evolution at each time step, grid point
  - Temperature, density from simulation
  - Initial abundances from small network
  - Burn each grid point for the simulation's time step
- Append results to original simulation data for visualization
- Evolve grid points in parallel with MPI

Conclusions and Future Work

- Cosmos++ now has:
  - Arbitrary network size support
  - Modernized reaction rates from StarLib reaction rate library
  - Massively parallel nucleosynthesis post processor
- Small networks can match energy generation but not isotopic resolution
- 46 isotope network is less accurate than 19 isotope network for twice the expense
- 136 is close to 495 but at much higher cost than 7/19
- Future Work
  - Use post processor on tidal disruption simulation
  - Compare expense and accuracy of small network + post processor against simulation with a large network
  - Quantify the uncertainty in nuclear energy generation from StarLib's included reaction rate uncertainties
  - Investigate GPU network evolution

References

[1] F. Timmes, R. Hoffman, and S. Wooseley, "An inexpensive nuclear energy generation network for stellar hydrodynamics," *ApJS*, vol. 129, pp. 377–398, 2000.

[2] P. Anninos, C. Fragile, and J. D. Salmonson, "Cosmos++: Relativistic magnetohydrodynamics on unstructured grids with local adaptive refinement," *AsPJ*, vol. 635, no. 1, 2005.

[3] A. Sallaska, C. Iliadis, A. Champange, S. Goriely, S. Starrfield, and F. Timmes, "Starlib: A next-generation reaction-rate library for nuclear astrophysics," *The Astrophysical Journal Supplement Series*, vol. 207, no. 1, p. 18, 2013.

[4] F. Timmes, "Integration of nuclear reaction networks for stellar hydrodynamics," *ApJS*, vol. 124, pp. 241–263, 1999.

[5] K. Kawana, A. Tanikawa, and N. Yoshida, "Tidal disruption of a white dwarf by a black hole," *MNRAS*, vol. 000, pp. 1–10, 2017.

Acknowledgments

I would like to thank my mentors Peter Anninos and Rob Hoffman for their help and advice. This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.

LLNL-POST-735903