Linear Stability Analysis of the Point Reactor Kinetics Equations with Feedback

ME262 Project Presentation

Samuel S. Olivier

December 11, 2018

Outline

1. Reactor Dynamics Background
2. Stability of Fuel Temperature Feedback
3. Stability of Coolant and Fuel Temperature Feedback

Reactor Dynamics Background

Point Reactor Kinetics Equations

- Describe the temporal behavior of a zero dimensional fission reactor
- Derived from time-dependent, one-group neutron diffusion equation with delayed neutron precursors
- Assumes power profile is separable in space and time
- Physics
  - Fission typically produces 2 or 3 neutrons/fission and 2 daughter products
  - Some daughter products (known as neutron precursors) decay by emitting a neutron
  - Typically model precursors by grouping them into 6 characteristic time scales

Point Reactor Kinetics Equations

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{\rho(t) - \beta}{\Lambda} P(t) + \sum_{i=1}^{6} \lambda_i C_i(t)\,,$$

$$\frac{\mathrm{d}C_i}{\mathrm{d}t} = \frac{\beta_i}{\Lambda} P(t) - \lambda_i C_i(t)$$

$P(t) =$ reactor power level

$C_i(t) =$ concentration of precursor i

$\rho(t) = \frac{k-1}{k} =$ reactivity (deviation from criticality)

$\beta_i =$ fraction of fissions that are born type i delayed, $\beta = \sum_i \beta_i$

$\Lambda =$ mean neutron generation time

$\lambda_i =$ decay constant for precursor i

Reactor Kinetics

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{\rho(t) - \beta}{\Lambda} P(t) + \sum_{i=1}^{6} \lambda_i C_i(t)\,,$$

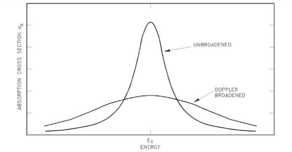
$$\frac{\mathrm{d}C_i}{\mathrm{d}t} = \frac{\beta_i}{\Lambda} P(t) - \lambda_i C_i(t)$$

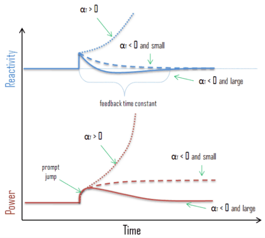
- Solutions of the form: $\sum_i A_i e^{s_i t}$
- Critical ($\rho = 0$)
  - Steady state
- Supercritical ($\rho > 0$)
  - Power grows exponentially
- Subcritical ($\rho < 0$)
  - Power decays exponentially

Reactor Dynamics

- In reality, feedback occurs
  - Fuel temperature increases → Doppler Broadening → more likely to be captured in non-fission reactions → lower power
  - Coolant temperature increases → density decreases → less effective moderator → less fission
- Changing power changes the reactivity of the system ⇒ non-linear

$$\rho(t) \rightarrow \rho(t, T_f(P), T_c(P))$$





Reactivity Feedback

$$\rho(t, T_f, T_c) = \rho_{\text{ext}}(t) + \alpha_f \Delta T_f + \alpha_c \Delta T_c$$

- $\rho_{\text{ext}}(t) =$ external reactivity addition
  - Moving control rods
  - Removing burnable poisons
- $\alpha_f = \frac{\partial \rho}{\partial T_f} =$ fuel temperature coefficient of reactivity
- $\alpha_c = \frac{\partial \rho}{\partial T_c} =$ coolant temperature coefficient of reactivity
- What values are required to damp out small reactivity perturbations?

Stability of Fuel Temperature Feedback

A Simple Feedback Model

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{\rho(t, T_f) - \beta}{\Lambda} P(t) + \lambda C\,,$$
$$\frac{\mathrm{d}C}{\mathrm{d}t} = \frac{\beta}{\Lambda} P(t) - \lambda C(t)\,,$$
$$\frac{\mathrm{d}T_f}{\mathrm{d}t} = K(P(t) - P_0)\ ,$$
$$\rho = \rho_{\mathsf{ext}}(t) + \alpha_f (T_f - T_{f0})$$

- Models “constant power removal” [2]
- One delayed neutron precursor group
- Can analytically show linear stability when $\alpha_f < 0$ by linearizing and using control theory

Stability Analysis Roadmap

- Linearize the governing equations without feedback
- Apply Laplace transform
- Solve algebraic system to find the response function $Z(s) : \rho \to P$
- Find the feedback response function $H_f(s) : P \to T_f$, $H(s) : T_f \to \rho$
- The response is of the form $\sum a_i e^{s_i t} u(t)$ where s_i are the poles of

$$T(s) = \frac{Z(s)}{1 - Z(s)H(s)}$$

- Stability occurs when $Re(s_i) < 0 \Rightarrow$ perturbations decay
- Problem of stability is replaced with finding the sign of the poles of the transfer function
  - Routh-Hurwitz Criteria: necessary and sufficient conditions for $Re(s_i) < 0 \ \forall i$

Linearizing the Feedback Model

- Linearize about steady state $\{P_0, C_0 = \frac{\beta}{\lambda\Lambda}P_0, T_{f0}\}$ with $\rho_0 = 0$
- $P = P_0 + \epsilon P'$, $C = C_0 + \epsilon C'$, $T_f = T_{f0} + \epsilon T_f'$, $\rho = \epsilon\rho'$
- Only non-linear term is:

$$\frac{\epsilon\rho'}{\Lambda}(P_0 + \epsilon P') \rightarrow \frac{P_0}{\Lambda}\rho'$$

- Linearized Equations:

$$\frac{\mathrm{d}P'}{\mathrm{d}t} = \frac{P_0}{\Lambda}\rho' - \frac{\beta}{\Lambda}P' + \lambda C'\,,$$

$$\frac{\mathrm{d}C'}{\mathrm{d}t} = \frac{\beta}{\Lambda}P'(t) - \lambda C'(t)\,,$$

$$\frac{\mathrm{d}T_f'}{\mathrm{d}t} = KP'\,,$$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{\rho(t,T_f)-\beta}{\Lambda}P(t) + \lambda C\,,$$
$$\frac{\mathrm{d}C}{\mathrm{d}t} = \frac{\beta}{\Lambda}P(t) - \lambda C(t)\,,$$
$$\frac{\mathrm{d}T_f}{\mathrm{d}t} = K(P(t) - P_0)$$

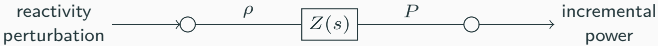
Laplace Transform

$$s\tilde{P} = \frac{P_0}{\Lambda}\tilde{\rho} - \frac{\beta}{\Lambda}\tilde{P} - \lambda\tilde{C}$$
$$s\tilde{C} = \frac{\beta}{\Lambda}\tilde{P} - \lambda\tilde{C}$$

- Laplace $\Rightarrow \frac{\mathrm{d}P'}{\mathrm{d}t} \rightarrow s\tilde{P}(s)$
- Turns system of ODEs into linear algebraic system
- Solve for response function: $\tilde{P} = Z(s)\tilde{\rho}$

$$Z(s) = \frac{s+\lambda}{s\Lambda\left(s+\lambda+\frac{\beta}{\Lambda}\right)}$$

- Shows how PRKEs without feedback will respond to small reactivity perturbations



Feedback Transfer Function

- How does the new perturbed power affect fuel temperature?

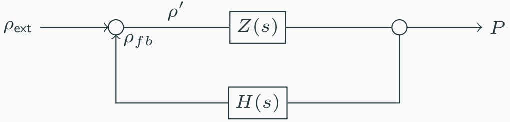
$$\tilde{T}_f = H_f(s)\tilde{P}$$

- Laplace Transform temperature equation:

$$s\tilde{T}_f = K\tilde{P} \Rightarrow H_f(s) = \frac{K}{s}$$

- Reactivity response from temperature change:

$$\rho_{fb} = \alpha_f T_f' \rightarrow \tilde{\rho}_{fb} = \alpha_f \tilde{T}_f = \alpha_f H_f(s)\tilde{P}$$
$$\Rightarrow H(s) = \frac{\alpha_f K}{s}$$



Closed Loop Transfer Function

- Transfer Function with Feedback

$$T(s) = \frac{Z(s)}{1 - Z(s)H(s)}$$

- Finding poles of $T(s)$ is equivalent to finding roots of characteristic polynomial

$$0 = 1 - Z(s)H(s) = 1 - \frac{s + \lambda}{s\Lambda\left(s + \lambda + \frac{\beta}{\Lambda}\right)} \frac{\alpha_f K}{s}$$

$$\iff \Lambda s^3 + (\lambda\Lambda + \beta)\, s^2 - \alpha_f K s - \alpha_f K \lambda = 0$$

- Must show $Re(s_i) < 0$

Routh-Hurwitz Stability Criterion

$$\Lambda s^3 + (\lambda\Lambda + \beta)\, s^2 - \alpha_f K s - \alpha_f K \lambda = 0$$

- Cubic polynomial of the form

$$a_0 s^3 + a_1 s^2 + a_2 s + a_3 = 0$$

- Routh-Hurwitz: all roots have negative real parts when
  1. $a_i > 0\, \forall i$
  2. $a_1 a_2 > a_0 a_3$
- $\Lambda, \lambda, \beta, K > 0 \Rightarrow a_2, a_3 > 0$ only when $\alpha_f < 0$
- $a_1 a_2 > a_0 a_3$ when

$$|\alpha_f|(\lambda\Lambda + \beta)\, K > |\alpha_f| \lambda\Lambda K$$

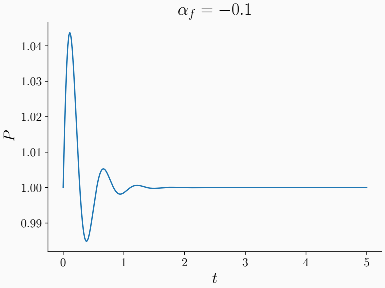
$$\Rightarrow \lambda\Lambda K + \beta K > \lambda\Lambda K$$

$$\iff \beta K > 0$$

  which is always true since $\beta, K > 0$
- Only stability requirement is: $\alpha_f < 0$

Theory and Simulation Agree on Stability

- Advance power, precursor, and temperature equations in time with Forward Euler
- Perturb the reactivity by 10¢ ($\frac{\beta}{10}$) and see whether the power returns to steady state



Stability of Coolant and Fuel Temperature Feedback

More Complex Feedback Model

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{\rho(t, T_f, T_c) - \beta}{\Lambda} P(t) + \lambda C\,,$$
$$\frac{\mathrm{d}C}{\mathrm{d}t} = \frac{\beta}{\Lambda} P(t) - \lambda C(t)\,,$$
$$\frac{\mathrm{d}T_f}{\mathrm{d}t} = KP - \gamma(T_f - T_c)\ ,$$
$$\frac{\mathrm{d}T_c}{\mathrm{d}t} = \frac{hA}{m_c C_c}(T_f - T_c) - \frac{2\dot{m}}{m_c}(T_c - T_{c,\mathrm{in}})\ ,$$
$$\rho(t, T_f, T_c) = \rho_{\mathrm{ext}}(t) + \alpha_f \Delta T_f + \alpha_c \Delta T_c$$

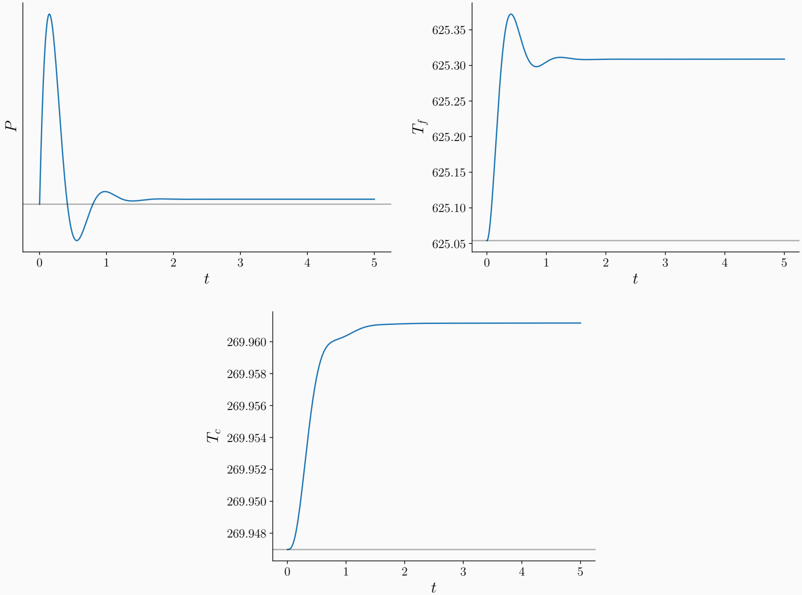
- Models heat transfer between the fuel and coolant/moderator
- Reactivity feedback from changes in fuel and moderator temperature
- Same linear stability + transfer function procedure $\rightarrow$ 4th order polynomial

Numerical Verification

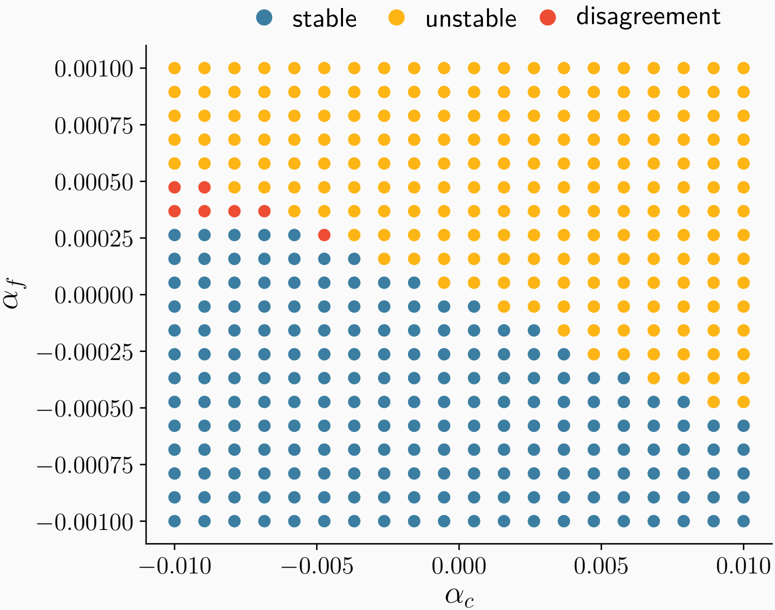
- Form analytic characteristic polynomial
- Determine stability programmatically with general Routh-Hurwitz (verifies $a_i > 0$ and the first column of the Routh Array is positive)
- Compare simulation to theory

Stable at New Equilibrium

$$\alpha_f = -1 \times 10^{-3}\,, \quad \alpha_c = -1 \times 10^{-3}$$



Parameter Search



Sources of Disagreement

- Theory predicted stability but simulation diverged
  - Forward Euler could be unstable, cranked the time step down, still diverged
- Problems for large, negative α_c and positive α_f
- Could be α_c is too large such that the feedback reactivity is not $O(\epsilon)$
- Non-linear effects?

Conclusions

- Analyzed two reactor dynamics models
  - Linearized the Point Reactor Kinetics Equations
  - Used Closed Loop Transfer Function to include effects of fuel and coolant temperature feedback
  - Found conditions for stability using Routh-Hurwitz Criteria
- Compared the linear stability theory results to simulation

References

[1] V. A. Kale, R. Kumar, K. Obaidurrahman, and A. Gaikwad, *Linear stability analysis of a nuclear reactor using the lumped model*, Nuclear Technology and Radiation Protection, 31 (2016), pp. 218–227.

[2] J. J. DUDERSTADT, *Nuclear Reactor Analysis*, John Wily & Sons, Inc., 1976.

[3] A. P. Guerrero, *Linear and Non-linear Stability Analysis in Boiling Water Reactors*, Woodhead Publishing, 2018.

[4] E. E. LEWIS, *Nuclear Reactor Physics*, Academic Press, 2008.

Questions?